Agitation of Viscous Newtonian and Non-Newtonian Fluids

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Viscous fluids are frequently agitated by multiple impellers and in vessels only slightly larger than the impeller. This paper presents data for both Newtonian and non-Newtonian fluids agitated under such conditions. The large decreases in power requirements (at a given level of mixing rate in the non-Newtonian system) which are possible by use of low tank diameter, impeller diameter ratios and/or two impellers, have been quantitatively studied.

The types of impellers used in the non-Newtonian work and the ranges of conditions over which power requirement correlations were developed are summarized as follows:

	D	T/D	n	N_{Re}
Marine propeller	0.42-1.0	1.4 -4.8	0.16-1.0	0.67-1320
Fan turbine	0.33-0.67	1.3 -3.0	0.21-1.4	6.6 -160
Flat-Bladed turbine:				
one impeller	0.17-0.67	1.3 -5.5	0.20-1.5	2.0 -1800
two impellers per shaft	0.33-1.00	1.023-3.5	0.14-1.00	0.15-620

The results generally confirm an approach developed earlier, for the broader ranges of variables listed above. For the non-Newtonian fluids of primary interest in this study, that is purely viscous materials having flow behavior indexes of less than unity (pseudoplastics, Bingham plastics), the prediction of power requirements has been developed to nearly the same level of perfection as for Newtonian fluids.

The purposes of this work were to extend earlier quantitative analyses (*10, 11*) of power requirements in agitation of non-Newtonian fluids and to obtain qualitative information on mixing rates in these systems. The several papers of similar scope which have appeared since our earlier work may be briefly reviewed as follows: Lee and co-workers (*7*) present data for viscous Newtonian fluids but were unable to achieve any correlation of their data on systems which were appreciably non-Newtonian. Foresti and Liu (*4*) correlated their data by an empirical modification of the Reynolds number term proposed for power law fluids (*11*). This does not represent a complete solution in that one must be able to define the shear rate range in which the power law constants are to be evaluated, hence recourse to reference *11* is necessary in any event. The paper by Calderbank and Moo-Young (*1*) extends the Metzner-Otto approach to several impeller types but was limited to very small mixers. In addition the value given for the proportionality constants of interest are, at least in one case, not in agreement with the slopes of the lines given in their paper and from which these proportionality constant were presumably derived. Further studies are clearly necessary.

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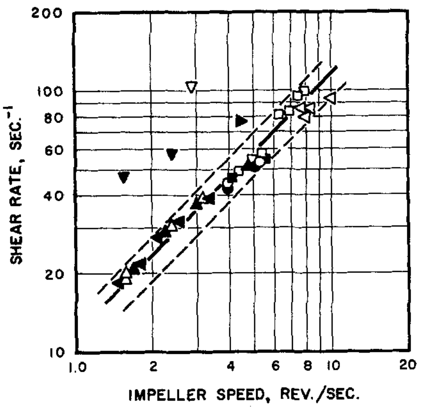
EXPERIMENTAL

In addition to equipment described previously (*11*) a 2-hp. mixer was used to enable the appreciable extension of the data under conditions of good mixing to agitated vessels as large as a 55-gal. drum. In the case of the 1/3-hp. mixer the power consumption was calculated from the measured motor speed and the reaction torque on the motor itself. As the larger motor was mounted in a fixed position, the torques were measured by placing the agitated tank upon a nearly frictionless table supported by radial and thrust bearings (*13*). It has been shown (*15*) that this arrangement is entirely satisfactory if the axis of the impeller is perpendicular to the torque table, provided of course the table remains nearly frictionless when the loaded tank is placed upon it.

Non-Newtonian slurries and polymeric solutions similar to those described in several earlier publications (*2, 11*) have been used. These fluids were chosen as typical of purely viscous (*9*) fluid systems, that is materials which, although non-Newtonian in shear, do not possess elasticity or any of the manifestations thereof to an appreciable degree. Such systems may be completely characterized by their shear stress-shear rate relationships under conditions of steady, simple shear, although they may not conform to the power law relationship between shear stress and shear rate so frequently applied to purely viscous fluids, and no assumption of this kind was required or employed in the present study. In addition to the solutions and slurries having flow behavior indexes below unity (pseudoplastics, Bingham plastics) concentrated plastisols of vinyl resins in di-octyl phthalate were employed to achieve highly dilatant behavior (flow behavior indexes greater than unity). These plastisols were similar to those described by Gunnerson and Gallagher (*6*).

Viscometric measurements made with capillary tubes of several L/D ratios to ensure absence of end effects were occasionally further checked with either a Couette or a cone-and-plate type of rotational viscometer. While no unusual problems were encountered, it was sometimes



SYMBOL	n	D	T/D
○	0.30	0.5	3.7
●	0.30	0.5	2.0
■	0.30	0.5	1.3
△	0.19	0.7	2.8
▲	0.19	0.5	3.6
◁	0.19	0.3	5.5
◀	0.19	0.5	1.3
□	1.5	0.3	3.0
▽	1.5	0.5	2.0
▼	1.5	0.7	1.5
▶	1.5	0.5	1.3

Fig. 1. Dependence of the mean shear rate upon rotational speed of the impeller (single flat-bladed turbine).

difficult to achieve shear rates low enough in the viscometric measurements on dilute solutions and suspensions and shear rates high enough in the dilatant systems. Accordingly measurements of agitator power consumption were discarded whenever they proved to be in a region of shear rates outside those of the viscometric measurements to ensure the absence of any errors which could be incurred by invalid extrapolation of the flow curves. The flow curves of the systems with flow behavior indexes less than unity were measured over at least the range of shear rates from 10 to 1,000 sec.$^{-1}$, and frequently much more, unless a smaller range covered the needs of the mixing studies. In dilatant systems shear rates from 10 to 300 sec.$^{-1}$ were usually obtained. Complete details are available in the original theses (*3, 8, 13, 16*).

RESULTS—POWER CONSUMPTION

Single Flat-bladed Turbines

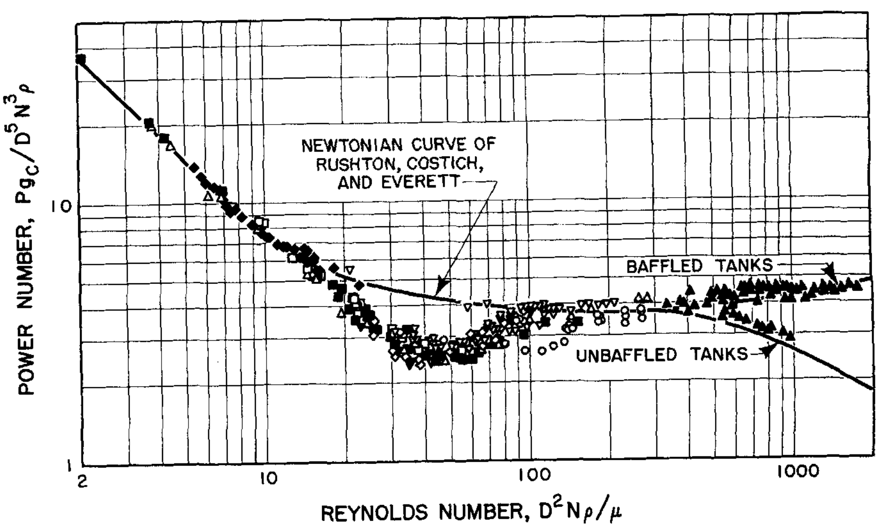
To check experimental techniques and accuracy the accepted power curve for agitation of Newtonian fluids (*14*) was reproduced between Reynolds numbers of 1.2 and 325, with a mean deviation of 5% or about the same accuracy as that with which the curve was originally derived. If one now determines the power number for a non-Newtonian fluid agitated under a given set of operating conditions a value for the mean viscosity of the fluid may be obtained by reading the corresponding Reynolds number from such a figure for Newtonian fluids and knowledge of the terms D, N, and ρ (*11*). Reference to the viscometric properties ($\mu_a = \tau g_c/u'$) of the fluid in turn enables the calculation of the mean shear rate under these conditions of fluid agitation. This analysis must be restricted to Reynolds numbers below about 10 for maximum sensitivity.

Figure 1 shows the shear rates as determined in this manner as a function of impeller speed. If the T/D ratio is not too close to unity one would expect the shear rate to be perfectly independent of both T and D by analogy to viscometric theory (*11*), as is found to be the case. The direct proportionality postulated earlier

$$u' = kN \qquad (1)$$

is seen to be obeyed by all data except those for dilatant fluids agitated in systems with low T/D ratios. The point value of the fluid shear rate increases very rapidly as one moves toward the impeller (*10*). This fact, together with the rapid increases of fluid viscosity with shear rate which occur in highly dilatant systems, suggests the near solidification of any dilatant fluid in the region of the impeller. The size of this nearly solid core increases rapidly with rotational speed and may be readily observed visually. The increasing importance of this core as the T/D ratio decreases is undoubtedly responsible for the abnormal increases in shear rate shown by Figure 1 for these systems. Unfortunately the data of Figure 1 are insufficient to define this effect completely in dilatant systems and indicate it to be far more important than the much weaker trends noted by Calderbank and Moo-Young in smaller vessels. Thus the present state of the art precludes the accurate prediction of power requirements in highly dilatant fluids at low T/D ratios.

For pseudoplastic fluids and for dilatant fluids in vessels with a T/D ratio of 3 or greater the solid line of Figure 1 gives $k = 11.6$ with a mean deviation of 8%, while all the correlated data fall within the ± 20% limits shown by the dashed lines. This is in



SYMBOL	FLUID	n
○	CMC	0.34
▽	ATTASOL	0.38
□	CARBOPOL	0.26
⊡	CARBOPOL	0.20-0.26
△	CARBOPOL	0.30-0.54
△	CARBOPOL	0.18-0.29
◇	PERMAGEL	0.16
▽	PERMAGEL	0.21
◈	PLIOVIC	1.5

Fig. 2. Power number—Reynolds number correlation for non-Newtonian fluids: single, flat-bladed turbine.

good agreement with the value of 13 developed earlier (*11*) with more limited data and the value of 10 with a maximum deviation of about 30% given by Calderbank and Moo-Young (*1*). The data of Foresti and Liu (*4*), kindly supplied in complete form by the authors, give $k = 11 \pm 5$. Actually for most fluids the choice of a value of k anywhere within this range is not critical; a 30% change in k results in only a 12% error in the viscosity of a fluid having a flow-behavior index of 0.5, for example.

Table 1. Ranges of Variables Covered in Study of Agitation by Two Turbines

Variable	Newtonian data	Non-Newtonian data
T	0.469-1.166 ft.	0.469-1.166 ft.
D	0.33-1.00 ft.	0.33-1.00 ft.
T/D	1.023-3.50	1.023-3.50
n	1.0	0.14-0.72
μ	1.48-184 poises	2.41-200 poises
N	0.03-16.8 rev./sec.	0.08-17.3 rev./sec.
N_{Re}	0.10-480	0.146-620
Power dissipation, hp./1,000 gal.	0.04-230	0.06-175

Baffles, used as indicated on the figures, were of a width equal to 1/10 T.

Figure 2 shows the correlation of 325 independent measurements with the numerical value of k in Equation (1) as 11.5* for purposes of defining the viscosity term in the Reynolds number. The data of references *1* and *4* are not included here as they would not serve to extend the ranges of variables covered except to lower Reynolds numbers which are of little interest in view of the very poor mixing under such conditions, and the scatter would have been increased appreciably. While the authors' data for dilatant fluids are included in Figure 2, neither these nor the other available literature data are for systems sufficiently dilatant to enable any clear distinction between the simple use of $k = 11.5$ and the more complex trends actually indicated by Figure 1. The ranges of variables covered by Figure 2 are

Impeller-diameters—	0.167-0.67 ft.
Tank diameters—	0.5-1.83 ft.
T/D ratios	1.3-5.5 (laminar region) 2.0-5.5 (transition region)
Power inputs—	0.4-176 hp./1,000 gal.
Impeller speeds—	1.58-29 rev./sec.
Reynolds numbers—	2-1760
Flow-behavior indexes—	0.2-1.5
Apparent viscosities—	1-180 poises

Both baffled ($J = 0.1T$) and unbaffled conditions were studied.

* That is the average of the values obtained with single and double impellers.

The depressive effect of pseudoplasticity upon insipient turbulence in the transition region is clearly shown by Figure 2. As this would be absent in dilatant systems, use of the Newtonian curve is recommended for these fluids, and a mean curve through the other data should be used for systems having flow-behavior indexes well below unity.

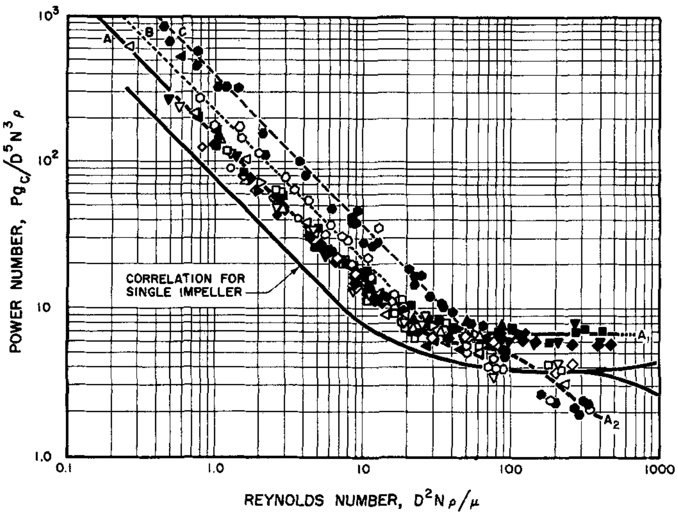
Multiple Turbines

In the light of data showing rapid decreases in mixing rates as one moves away from the immediate region of the impeller (*10*) it is obvious that to achieve good mixing rates everywhere in the tank with minimum power consumption the T/D ratio should be as low as feasible and the use of more than one impeller may be desirable, especially in fluid systems having low flow-behavior indexes.* This requires extension of the available Newtonian data as well, as the best available data only go down to a T/D ratio of 1.4 and very few measurements exist below a T/D ratio of about 2.0 (*14, 17*), except for anchor types of agitators.

The following data were obtained by affixing two identical flat-bladed turbines to the same shaft, separated by one half a tank diameter. The fluid depth was always equal to the tank diameter, and the lower turbine was located between 16 and 20% of a tank diameter from the bottom of the vessel. This arrangement of the impellers appeared to be optimum for rapid mixing.

Figure 3 shows that in the laminar region two impellers draw (within a few percent) twice the power of a single impeller provided the T/D ratio remains above about 1.25. This is in agreement with the only available prior art data (*7*). As the T/D ratio drops below this level, wall effects cause additional increases in power consumption.

Wall effects would be less important in fluids with low flow-behavior indexes, since decreases in viscosity accompany



SYMBOLS, T/D RATIOS					
D \ T	1.166	0.786	0.698	0.682	0.496
1.000	1.166 ○	-----	-----	-----	-----
0.666	1.75 ○●	1.18 ○	1.048 ●	1.023 ●	-----
0.500	2.33 △▲	1.57 □■	1.39 ◁	-----	-----
0.333	3.50 ▷▶	2.36 ◇◆	2.10 ▽▼	-----	1.41 ◀
●▼▲■◆ BAFFLED TANK ○□○◇▽△○◁ UNBAFFLED TANK					

Fig. 3. Power number—Reynolds number correlation for Newtonian fluids: *two flat-bladed turbines*: Curve A-A_1 : $T/D > 1.25$, baffled tanks, A-A_2 : $T/D > 1.25$, unbaffled, Curve B : $T/D = 1.16$-1.18, C : $T/D = 1.02$-1.05. Curves B and C join curves A-A_1 and A-A_2 as the Reynolds numbers increase.

* This statement presupposes that the intense mixing rates obtainable near a small impeller operating at a high speed are not needed. This appears to be the case in many mixing operations, but some others, especially emulsification processes, require them. A higher T/D ratio may be necessary in those instances.

any increases in shear rate. In these systems it may be expected that the T/D effects of Figure 3 might be absent. Preliminary analysis of the data showed this to be the case for all the highly non-Newtonian ($n < 0.70$) systems studied. The dependence of the mean shear rate upon impeller speed, calculated on this basis, is given in Figure 4. The value obtained for k is 11.4, with a mean deviation of 16%, in excellent agreement with Figure 1.

Figure 5 gives the power number—Reynolds number plot for the pseudoplastic fluids. The ranges of variables covered by the 445 non-Newtonian measurements of Figure 5 and the 303 Newtonian data of Figure 3 are given in Table 1.

No correlation of data on dilatant systems is presented, as these would normally not be processed by use of multiple impellers.

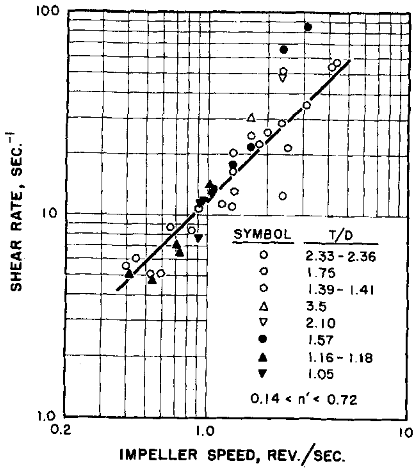


Fig. 4. Dependence of shear rate on impeller speed, two flat-bladed turbine impellers.

Fan Turbine

Data similar to Figures 1 and 4 yielded a value of 13 ± 2 for the constant of Equation (1) for turbines having six flat blades (length: width ratio = 1.25) affixed to a hub at a 45-deg. angle from the vertical, when agitating pseudoplastic fluids. Figure 6 shows the resultant power number—Reynolds number plot for dilatant materials as well as for fluids with flow behavior indexes below unity. The ranges of variables covered (including the Newtonian points not shown in Figure 6) are

	$n < 1$	$n = 1$	$n > 1$
T, ft.	0.67-0.98	0.67-0.98	0.67-0.98
D, ft.	0.33-0.67	0.33-0.50	0.33-0.67
T/D	1.33-3.0	1.33-3.0	1.33-3.0
n	0.21-0.26	1.00	1.0-1.42
μ, poises	0.5-3.2	1.9-11	1.9-3.2
N, rev./sec.	2.8-26	1.5-15	1.7-16
N_{Re}	6.6-160	1.1-43	7.9-33
Power dissipation hp./1,000 gal.	8.7-160	7.5-98	5.9-130
No. of independent measurements	46	35	49

Both baffled ($J = 0.1$ T) and unbaffled conditions were studied; within the ranges covered no effect on power consumption was observed. The Newtonian curve is in good agreement with the prior art (*14*). As would be expected the dilatant data fall on the Newtonian curve in the transition region, and the transition to turbulence is delayed in fluids of low flow-behavior index. The correlation cannot be recommended for dilatant systems beyond the ranges covered.

Marine Propellers

Both single- and double-pitch, three-bladed marine propellers were used in this study. The effect of impeller position was evaluated under two conditions: shaft vertical at vessel center line, vessel unbaffled; shaft 10 deg. from vertical, displaced a distance equal to $R/3$ from center as shown in Figure 7. This position was determined by agitating Newtonian fluids and observing conditions under which power input could be maximized without causing air entrainment (that is this position represented an effective type of baffling).

The fluid depth equaled the tank diameter, propeller immersion was equal to 50% of the fluid depth, and the fluid was displaced upwards as well as downward by using reverse- as well as forward-pitch propellers. The data taken on Newtonian fluids (*3*) agreed well with the results of Rushton et al. (*14*).

Figure 8 shows that the proportionality constant between shear rate and impeller speed is equal to 10 for marine propellers agitating fluids with flow-behavior indexes below unity, irrespective of shaft position, pitch, or direction of fluid displacement. The mean and maximum deviations of the points from this value are 7 and 20%, respectively.

In highly dilatant systems the aforementioned core of nearly solidified material around the impeller was again observed. The sheared region between the edge of this core and the walls of the tank becomes narrower as impeller speeds are increased, and more than a merely proportionate increase in shear rate, especially at low T/D ratios, would be expected. Figure 9 shows some limited data obtained in very highly dilatant systems having

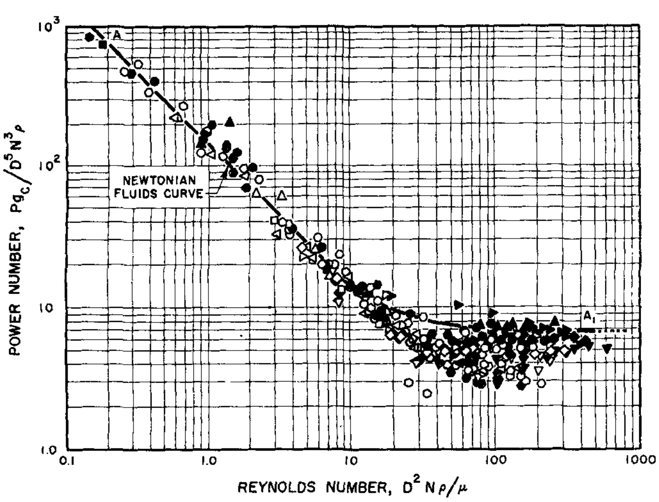


Fig. 5. Power number—Reynolds number correlation for non-Newtonians—two flat-blade turbine impellers. See Figure 3 for legend.

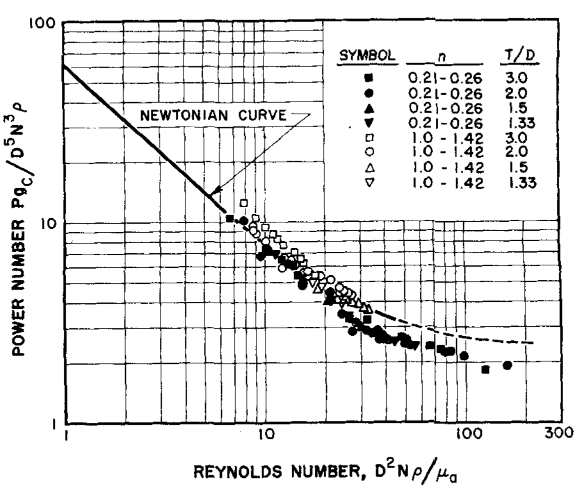


Fig. 6. Power number—Reynolds number correlation for a six-bladed fan turbine.

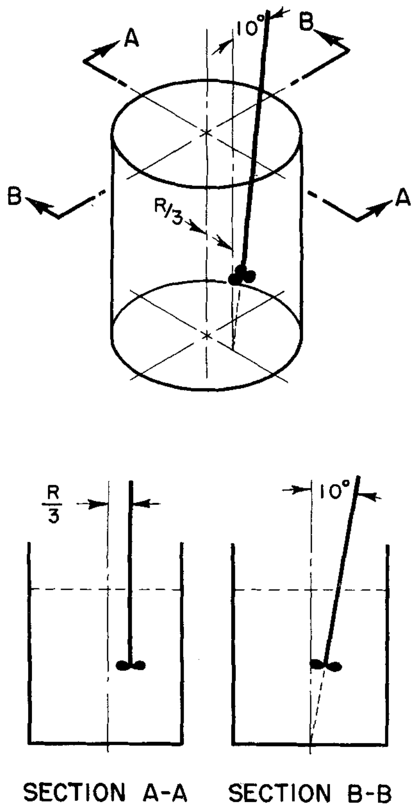
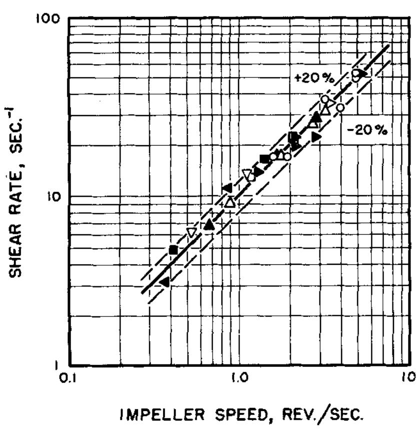


Fig. 7. Description of shaft position II used with the marine propellers.

flow-behavior indexes between 1.3 and 2.8 at the shear rates of interest. All of the lines have been drawn with a slope equal to 2.0; while this may not prove to be entirely correct in the light of more extensive data, there can be no question about the fact that the shear rates generally increased more nearly as the square of the impeller speed than as the first power. Since these data are for lower T/D ratios and for more highly dilatant fluids than those shown in Figure 1, the stronger effects of both T/D and impeller speed appear quite reasonable. While the curves of Figure 9 may be used to correlate the power requirements measured in the present study, it is felt that more information is required before such a correlation is of proven value for design purposes.

Figures 10 and 11 present power number-Reynolds number correlations for the propellers over the entire ranges of variables studied with fluids of low flow-behavior index. It is seen that not only is the transition to turbulence somewhat delayed in the non-Newtonian systems (Figures 2 and 6) and that baffling (shaft position) is important, but that in addition a definite effect of impeller diameter exists. This is in no way due to the method chosen for evaluation of viscosity, as the effect persists at higher Reynolds numbers where viscosity has ceased to be a primary variable. Since the causes of this effect are not entirely resolved by Figures 10 and 11, the more conservative curves should be used when extrapolations to larger systems are made.

The ranges covered by Figures 10 and 11 for those variables not defined on the figures themselves are



SYMBOL	DIAMETER	T/D	POSITION	PITCH
○	0.417'*	2.27	I	SINGLE
●	0.417'*	2.27	II	
▷	0.417'*	2.74	I	
▶	0.417'*	2.74	II	
▽	1.00'	1.94	I	
◀	1.00'	1.94	II	
◁	8"	1.41	I	DOUBLE
△	8"	2.28	I	
▲	8"	2.28	II	
□	8"	2.91	I	
■	8"	2.91	II	

Fig. 8. Dependence of mean shear rate upon impeller speed: *marine propellers in non-Newtonian* ($n < 1$) fluids. Diameter terms followed by an asterisk indicate fluid displaced upwards through the impeller; in all other cases the fluid was displaced downwards.

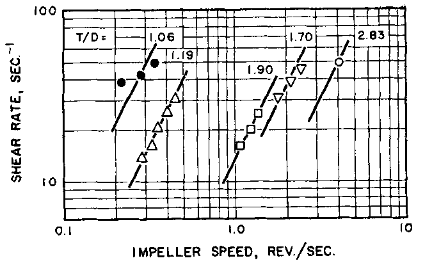
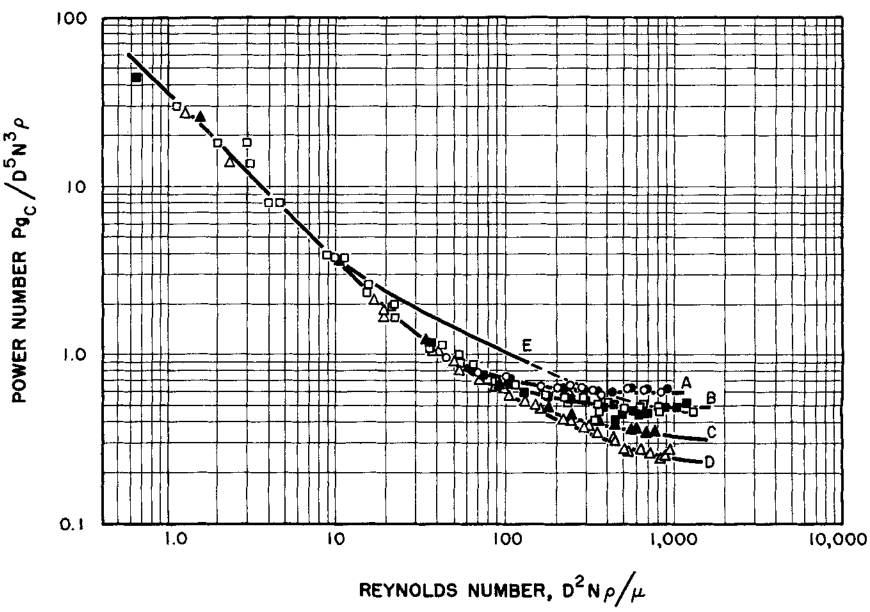


Fig. 9. Shear rate—impeller speed relationships for propellers in highly dilatant fluid systems (tentative).



CURVE	D	T/D	SHAFT POSITION
A	0.417	2.2 - 4.8	I, II
B	0.417*	2.2 - 4.8	I, II
C	1.00	1.9 - 2.0	II
D	1.00	1.9 - 2.0	I
E	NEWTONIANS		II

Fig. 10. Power number—Reynolds number correlation for square-pitch propellers ($n < 1.0$). Asterisks denote upward displacement of fluid.

Flow behavior indexes	0.16 to 0.40
Viscosity, poises:	0.09-14.4

MIXING RATES

Table 2 compares the Reynolds numbers at which the contents of an agitated vessel first showed movement at the surface of the vessel. This is a good index of the minimum agitation rate required to obtain some mixing in all portions of a tank, since the fluid at the surface is usually the last to show any motion, except for very small regions in the neighborhood of any baffles.

Noting that two turbines draw nearly twice the power of a single one, one can see that at very low T/D ratios there is little or no advantage to the use of multiple impellers. However at the higher T/D ratios (above about 2.0) the power required to turn over the entire contents of the vessel may be reduced by as much as an order of magnitude by using two impellers. Furthermore the system of two impellers apparently continues to recirculate nearly twice as much fluid as a single turbine, since the power number remains almost twice as high within the whole range of Reynolds numbers studied. This indicates that the mixing rates at all Reynolds numbers would appear to remain at least as high as for a single turbine, at a given power input. This argument is supported by an investigation of rates of fast reactions in agitated vessels (*12*), where it was shown that the rate appears to be completely controlled by this rate of gross fluid turnover. Therefore unless one is carrying out a process in which the high shear rates in the region of a single impeller (at the higher Reynolds numbers, hence power consumption rates needed to put the entire contents of the vessel into motion) are of particular importance to the process, as in the case of an emulsification or mass transfer processes, for example (*12*), the use of multiple impellers would appear to be of general interest in viscous non-Newtonian fluid systems.

The data given in Table 2 are very limited in the case of propellers. However they indicate a general undesirability of using this type of agitator in viscous non-Newtonian fluid systems, in agreement with the prior art (*12*) except possibly at low T/D ratios.

The data for the fan turbine show a very marked superiority over the other two impellers and even over the system of two flat-bladed turbines except at the highest T/D ratios. This apparent advantage is largely due to the fact that much difficulty is encountered in getting the last 10 to 15% of the fluid in motion when flat-bladed turbines are used. The axial flow created by the fan turbine overcomes this problem very effectively. However this advantage of the fan turbine may be expected to be greatly reduced at higher mixing rates when the entire contents of the vessel are in good motion in any event.

In dilatant systems turnover of the entire contents of the vessel was found to require Reynolds numbers of about 10 to 20 in the case of flat-bladed turbine impellers. As would be expected from a consideration of fluid properties this level was insensitive to both the number of impellers and the T/D ratio. This level of the Reynolds numbers agrees well with those of Table 2 in the sense that complete fluid turnover in all cases occurs beyond the end of the laminar region as defined by the power number—Reynolds number curves.

The only prior-art information on non-Newtonian fluid agitation rates to which the above results may be compared appear to be the industrial data cited in reference (*11*). Those data agree well with the results for a single turbine in this work; the industrial usage of up to between 50 and 160 hp./1,000 gal. can be appreciably reduced by employing multiple impellers unless viscosities are higher than the 200 poise maximum of the present study. References 5 and *12* review the available mixing rate studies in Newtonian systems.

SUMMARY, CONCLUSIONS, AND RECOMMENDED DESIGN PROCEDURE

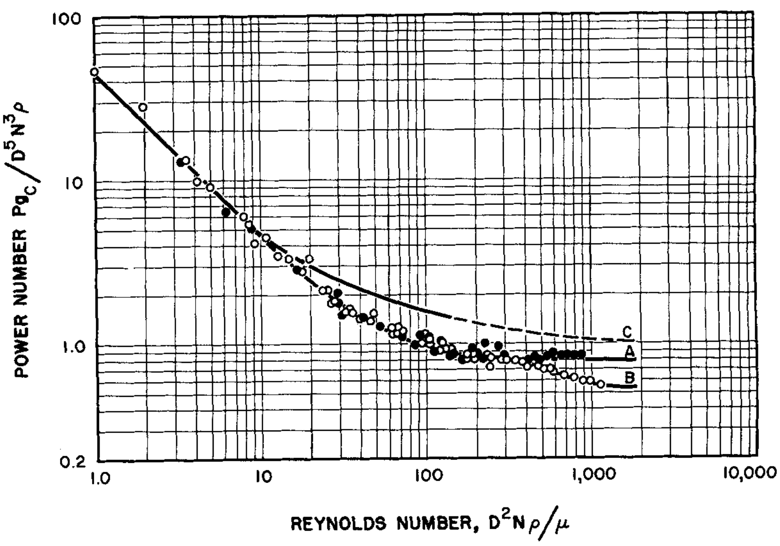
1. Power requirements for the agitation of viscous Newtonian fluids have been extended to systems employing two turbines and to low T/D ratios. These results are shown in Figure 3.

2. The direct proportionality between the mean fluid shear rate and the impeller speed, independent of all other variables as postulated earlier, has been confirmed for flat-bladed turbines and extended to fan turbines and marine propellers. The proportionality constant was found to have the following values:

Flat-bladed turbines:	$k = 11.5 \pm 1.4$
Fan turbine	$k = 13 \pm 2$
Marine propellers (single and double pitch) :	$k = 10 \pm 0.9$

Thus for most practical purposes a universal value of about 11 suffices for all three impellers. As an approximation its use with other impeller types is also suggested.

These conclusions are applicable only to purely viscous fluids having flow-behavior indexes of less than unity



CURVE	T/D	SHAFT POSITION
A	1.4 - 3.0	II
B	1.4 - 3.0	I
C	NEWTONIANS	II

Fig. 11. Power number—Reynolds number correlation for double-pitch propellers ($n < 1.0$).

TABLE 2. MINIMUM REYNOLDS NUMBER REQUIRED FOR MOVEMENT OF SURFACE FLUID IN NON-NEWTONIAN SYSTEMS ($n < 1.0$)

T/D	Single turbine	Two turbines	Fan turbine	Vertical propeller	Tilted propeller
4.8				640	430
3.5	>300	90			
3.0	270		120		270
2.4		75			
2.3				320	60
2.1	160	70			
2.00	110		50		
1.75		50-55			
1.50	90			200	
1.40		40-45			
1.33	50		30		
1.17		40-45			
1.05		35			
1.02		35			

(pseudoplastics, Bingham plastics) although the restriction of purely viscous behavior may not be of primary importance at the modest shear rates generally encountered in this type of mixer. While preliminary data on dilatant systems indicate much more complex relationships, the above recommendation will also serve as a fair approximation in these systems unless the flow-behavior index is unusually high or the T/D ratio low.

3. With the above shear rate-impeller speed dependency, power number-Reynolds number correlations for non-Newtonian systems are presented in Figures 2, 5, 6, 10, and 11 for single and double turbines, fan turbines, and marine propellers. The primary limitation of these data lies in the fact that the maximum impeller diameter used in this work was 1 ft. Generally this is not likely to be as serious a limitation as it would be in Newtonian systems, since the high level of power consumption of the non-Newtonian fluids limits equipment size, and considerable extrapolation is permissible in view of the ranges of diameters covered and confirmation of the basic hypothesis by a photographic study of fluid motion (*10*). With this possible exception it is now possible to predict the power requirements of these mixers agitating non-Newtonian systems with the same accuracy as for Newtonian fluids. The data have not been extended to extremely high Reynolds numbers because the very presence of appreciable non-Newtonian characteristics implies high viscosity levels. Again highly dilatant systems are excluded but will only rarely be encountered.

4. The distinct advantages in terms of agitation rate at a given level of power consumption which are to be gained by use of multiple turbines have been defined. These advantages become more important as the T/D ratio increases.

The fan turbine possesses a clear advantage over flat-bladed turbines at low agitation rates. At higher agitation rates this advantage will probably be lost to a large degree. In these viscous non-Newtonian systems the marine propeller may be a less useful type of impeller than either turbine, although the data are not entirely conclusive on this point.

5. The recommended design procedure may be summarized as follows: From a knowledge of the desired mixing rate the impeller size and speed are determined. Table 2 may be used as a guide to the minimum conditions required for complete mixing. If no revelant unpublished data or prior experience exist with which the impeller speed may be estimated at high mixing rates, the correlations for Newtonian systems (*5*, *12*) may be used as good approximations in most cases.

When one knows the impeller speed and the rheological properties of the fluid being agitated, the viscosity at a shear rate equal to k times the impeller speed is calculated.

The numerical value of the Reynolds number may now be computed and reference to Figure 2, 5, 6, 10, or 11 gives the power number, hence the shaft horsepower required.

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NOTATION

- D = impeller diameter, ft.
- g_c = dimensional conversion factor, 32.2 (ft.)(lb.$_M$)/(sec.^{2})(lb.$_F$)
- J = baffle width, ft.
- k = dimensionless proportionality constant of Equation (1)
- n = flow-behavior index of a non-Newtonian fluid, dimensionless (see references 9 and *11* for example)
- N = rotational speed of impeller, rev./sec.
- N_p = power number, dimensionless, $N_p = \rho g_c/D^5N^3\rho$
- N_{Re} = Reynolds number, dimensionless $N_{Re} = D^2N\rho/\mu$ or $D^2N\rho/\mu_a$
- P = power consumption, (ft.)(lb.$_F$)/(sec.)
- R = tank radius, ft.
- T = tank diameter, ft.
- u' = shear rate, sec.$^{-1}$
- μ = viscosity, lb.$_M$/(sec.)(ft.), μ_a is sometimes used to emphasize that the viscosity (or apparent viscosity) of a non-Newtonian fluid is a function of shear rate
- ρ = density lb.$_M$/cu. ft.
- τ = shear stress, lb.$_F$/sq. ft.

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